RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [2017-20]
B.A./B.Sc. THIRD SEMESTER (July – December) 2018
Mid-Semester Examination, September 2018

Date : 26/09/2018 **MATH FOR ECONOMICS (General)**
Time : 12 noon – 1 pm Paper : III Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Answer **any two** from **Question Nos. 1 to 4** : [2×4]

1. For the function $f(x,y)=\begin{cases}(x^2+y^2)\log(x^2+y^2), & \text{for}(x,y)\neq(0,0)\\ 0 & ,\text{for}(x,y)=(0,0)\end{cases}$

   Show that Schwarz's theorem condition's are not satified but $f_{xy} = f_{yx}$ at (0,0) [2+2]

2. a) State Euler's theorem for homogenous function of two variables. [1]

   b) Let, $u=\text{Sin}^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ .Prove that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=-\dfrac{\text{Sin}\,u\,\text{Cos}\,2u}{4\cos^3 u}$ [3]

3. a) Are the functions u= x+y-z , ν = x-y+z, w= $x^2+y^2+z^2 - 2yz$ are functionally related? [2]

   b) Let, f(x,y) = $|x|+|y|$, check the differentiability of the function f at (0,0) [2]

4. Find the minimum value of $x^2+y^2+z^2$ subject to ax+by+cz = p. [4]

Answer **any one** from **Question Nos. 5 to 6** : [1×5+]

5. Suppose the marginal rate of substitution of good 1 for good 2 is $\dfrac{x_1}{x_2}$.

   a) Derive the equation of the indifference curve ? What is the utility function associated with it. What if the marginal rate of substitution of good 1 for good 2 is K > 0 ? $[1\frac{1}{2}+\frac{1}{2}+1]$

   b) Test for the concavity or quasi-concavity of the utility functions that are obtained in the part(a). [1+1]

6. a) i) Prove that diminishing marginal utility is neither necessary nor sufficient for diminishing marginal rate of substitution.

   OR

   ii) Show that the CES production function $F(L,K)=A\left[\alpha L^{-\rho}+(1-\alpha)K^{-\rho}\right]^{-\frac{1}{\rho}}$ where A > 0 and $0 < \alpha < 1$,approaches to the Cobb-Douglas production function G(L,K) = $AL^{\alpha}K^{1-\alpha}$ when $\rho\to 0$ [3]

   b) Find the directional derivative of the function f(x,y) =$x^2y^3 - 4x$ at the point (3,1) in the direction ν =2i+5j where i = $\begin{pmatrix}1\\0\end{pmatrix}$ and j = $\begin{pmatrix}0\\1\end{pmatrix}$. [2]

Group – B

Answer question no. 7 and **any two** from **Question Nos. 8 to 11**: [2+2×5]

7. Determine the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)=5\left\{1+\left(\frac{d^2y}{dx^2}\right)^2\right\}^{\frac{1}{4}}$ [1+1]

8. a) Define a first order homogeneous differential equation verify, whether the following differential equation is a homogeneous differential equation $\frac{dy}{dx}=3\log(x+y)-\log(x^3+y^3)$ [1+2]

   b) Solve $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ [2]

9. a) Define an exact differential equation of order one .State the necessary condition for the ordinary differential equation M(x,y)dx + N(x,y)dy =0 to be an exact differential equation. [2+1]

   b) Show that $\frac{1}{x^5}$ is an integrating factor of the differential equation $(x^4-y^4)\,dx- xy^3dy=0$ [2]

10. Solve: $\left(x+2y^3\right)\frac{dy}{dx}=y$ [5]

11. Find the general and singular solution of the differential equation $y=px+p-p^2$, where $p=\frac{dy}{dx}$ [2+3]

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